Paper ID:MT596 The 3rd International Conference on Mechatronics and Automation Technology (ICMAT 2024) October 25-27, 2024, Wuhan, China

Position Tracking for a Series of Mass-Spring-Damper System via Decentralized Control Design

Zheng-Ji Mao, School of Ocean Mechatronics, Xiamen Ocean Vocational College, Xiamen, China

E-mail: maozhengji@xmoc.edu.cn

Introduction: In this paper we present a decentralized stabilization control technique for linear time-varying interconnected systems. By the optimal control technique with respect to a performance index, we have developed the subsystem based time-varying feedback law decentrally. Since the considered system is time-varying, the time-varying Riccati differential equations need to be solved. The backward Euler's method is used to obtain solutions from a sufficient large time to the initial time with a guessed terminal values. When the initial conditions are obtained, these Riccati equations will be introduced into the overall system for the implementation.

Model description: A series of masses

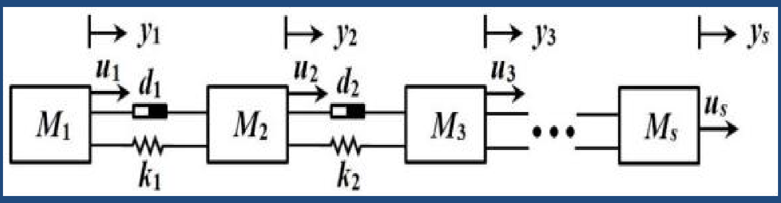

Figure 1. A series of mass-spring-damper systems

Model equations:

When *i* = 2, 3, ..., *s*–1, the generalized equation is

$$M_i\ddot{y}_i + (d_{i-1}+d_i)\dot{y}_i + (k_{i-1}+k_i)y_i = d_{i-1}\dot{y}_{i-1} + k_{i-1}y_{i-1} + d_i\dot{y}_{i+1} + k_i y_{i+1} + u_i$$

The last mass, the equation is

$$M_s\ddot{y}_s + d_{s-1}\dot{y}_s + k_{s-1}y_s = d_{s-1}\dot{y}_{s-1} + k_{s-1}y_{s-1} + u_s$$

The matrix form:

$$\dot{x}_i = \begin{bmatrix}\dot{x}_{i1}\\ \dot{x}_{i2}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\dfrac{k_{i-1}+k_i}{M_i} & -\dfrac{d_{i-1}+d_i}{M_i}\end{bmatrix}x_i + \begin{bmatrix}0\\ \dfrac{1}{M_i}\end{bmatrix}\left(u_i + \begin{bmatrix}k_{i-1} & d_{i-1}\end{bmatrix}\begin{bmatrix}x_{(i-1)1}\\ \dot{x}_{(i-1)2}\end{bmatrix} + \begin{bmatrix}k_i & d_i\end{bmatrix}\begin{bmatrix}x_{(i+1)1}\\ \dot{x}_{(i+1)2}\end{bmatrix}\right)$$

$$= A_i x_i + B_i\left(u_i + \begin{bmatrix}k_{i-1} & d_{i-1}\end{bmatrix}x_{i-1} + \begin{bmatrix}k_i & d_i\end{bmatrix}x_{i+1}\right),\quad i = 1,\ 2,\ \ldots,\ s$$

$$\dot{x}_i = A_i x_i + B_i\left(u_i + \sum_{j=1}^{s} q_{ij}x_j\right)$$

The Decentralized Control Design:

$$\dot{x}_d = Ax_d + Bu_d + BQx_d = 0$$

$$u_d = -(B^T B)^{-1}B^T(A+BQ)x_d$$

$$\dot{x} = \dot{\tilde{x}} + \dot{x}_d = \left(A+BQ\right)(\tilde{x}+x_d) + B(\tilde{u}+u_d)$$
$$= \left(A+BQ\right)\tilde{x} + B\tilde{u} + \left(A+BQ\right)x_d + Bu_d$$

Computer simulations:

A 4-mass interconnected mass-spring-damper system is considered and the parameters are as follows.

$$M_1 = M_2 = M_3 = M_4 = 1;\ \ d_1 = 0.1,\ d_2 = 0.2,\ d_3 = 0.3;$$

$$k_1 = 1,\ k_2 = 1.2,\ k_3 = 1.3;\ \lambda = 1$$

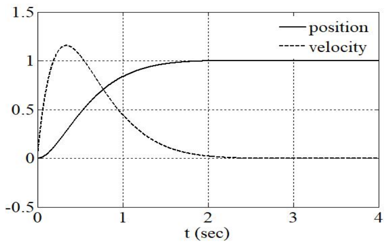

Figure 2. Responses of mass-1

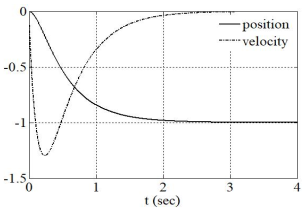

Figure 3. Responses of mass-2

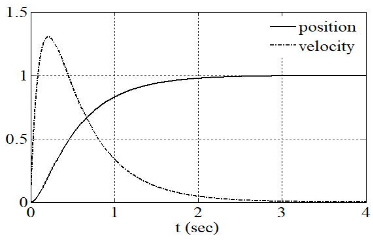

Figure 4. Responses of mass-3

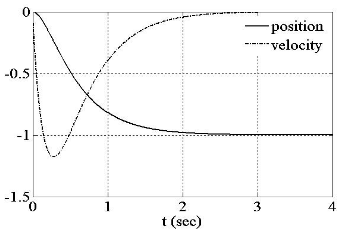

Figure 5. Responses of mass-4

Conclusion: The mass-spring-damper system is very common in the application field and many systems can have the similar properties. In this paper we first make an interconnection model and then derive a decentralized control design to make each mass approaches its reference exponentially.

References

[1] Carli R, Dotoli M. A dynamic programming approach for the decentralized control of energy retrofit in large-scale street lighting systems. IEEE Transactions on Automation Science and Engineering, 2020: 1140-1157

[2] Biswal S, Elamvazhuthi K, Berman S. Decentralized control of multi-agent systems using local density feedback. IEEE Transactions on Automatic Control., 2022, doi: 10.1109/TAC.2021.3109520

[3] Lin W, Bitar E. Decentralized Control of Constrained Linear Systems via Assume-Guarantee contracts American Control Conference, 2020: 917-924

[4] Liu T, Jiang Z P, Zhang P. Decentralized event-triggered control of large-scale nonlinear systems. International Journal of Robust and Nonlinear Control, 2020: 1451-1466

[5] Rugh W J. Linear system theory. Prentice-Hall Inc, 1993