Paper ID: ICMAT2024-571 The 3rd International Conference on Mechatronics and Automation Technology (ICMAT 2024)

Improved Decision Making in Mechanical Repairable Systems with Preventive Maintenance through Bayesian Analysis

Anxiong Wu and Chih-Chiang Fang
Dep. of Computer Science of ZhaoQing University, ZhaoQing 526061, China

INTRODUCTION: As systems and their components age, they naturally deteriorate, making preventive maintenance a crucial strategy for maintaining system functionality and extending operational lifespan. Preventive maintenance not only restores the system but also enhances its overall health, effectively slowing the aging process. While significant research has focused on identifying optimal preventive maintenance policies for repairable systems, these decisions often encounter considerable uncertainty, compounded by the challenge of limited data. Therefore, maximizing the use of available information becomes essential. In this paper, we introduce a Bayesian decision model designed to optimize the number of preventive maintenance actions for systems maintained through periodic preventive maintenance policies. The system's deterioration is modeled using a non-homogeneous Poisson process with a power-law failure intensity. The model assumes that after each preventive maintenance action, the system's condition lies between "as good as new" (perfect repair) and "as good as old" (minimal repair). Failures occurring between maintenance intervals are addressed through minimal repairs. A numerical method and solution algorithm are provided to demonstrate the proposed approach for mechanical engineers in practice.

MODEL DEVELOPMENT: To model the system's deterioration process, a Weibull power law intensity function $\lambda(t)=\alpha\beta t^{\beta-1}$ is applied within the non-homogeneous Poisson process (NHPP) framework. This function is chosen for its flexibility and practical application in capturing the system's aging behavior. In this formula, α serves as the scale parameter, representing the system's rate of deterioration, while β is the shape parameter, reflecting how the failure rate evolves over time. The variable t denotes the elapsed time. Figure 1 presents a timeline that visually represents this periodic PM model, showing the intervals between PM actions, the system's operational states, and the eventual replacement at the N-th PM action.

PREVENTIVE MAINTENANCE SCHEME:

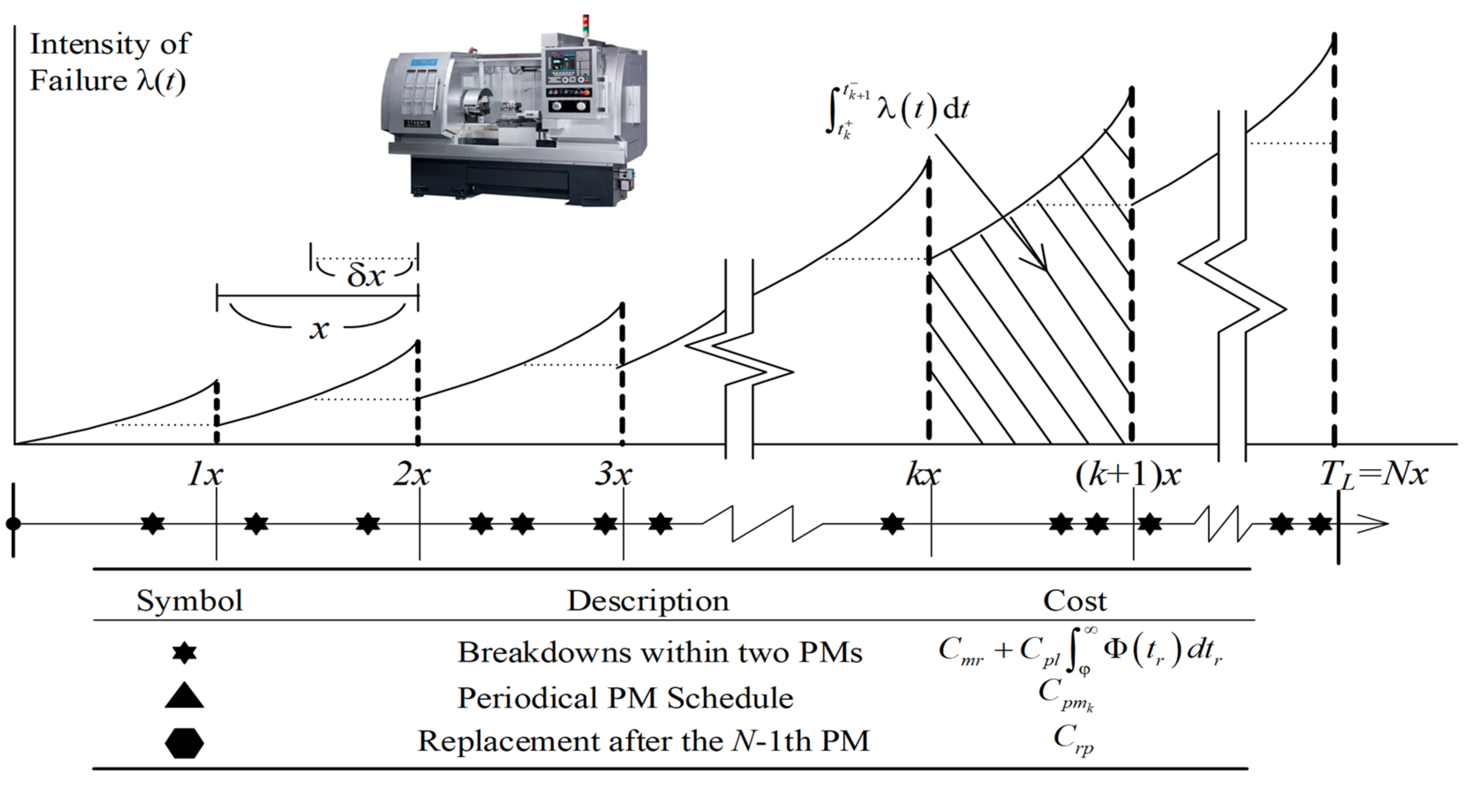

Figure 1. Timeline of Preventive Maintenance of a Machine.

THE ANALYSIS USING A NATURAL CONJUGATE PRIOR: Bayesian decision analysis can be quite challenging due to the intricate process of deriving posterior distributions, which often requires numerical integration techniques. This complexity becomes particularly evident in cases like the current study, where two random variables, α and β, form the state space. These variables represent key parameters governing the system's degradation and failure dynamics, and calculating their posterior distributions is typically computationally demanding. The advantage of using this conjugate prior is that it simplifies the Bayesian analysis, eliminating the need for complex numerical integration. The natural conjugate prior distribution is expressed as:

$$f(\alpha,\beta) = K\alpha^{b-1}\beta^{b-1}\left(e^{-a}d^{b}\right)^{\beta-1}e^{-\alpha c d^{\beta}}$$

By using equation (8) as the prior distribution for α and β, the prior mean of interest can be easily derived, given by:

$$E\left[\alpha t^{\beta}\right] = \frac{ba^{b}}{c}\left[(a+\ln d-\ln t)^{-b}\right].$$

Thus, the expected total cost per unit time in the prior analysis within the system's life T_L can be rewritten as follows:

$$E[C(N)] = \sum_{i=1}^{N-1} C_{pm_i} + C_{rp} + \left(C_{mr} + C_{pl}\int_{\varphi}^{\infty} t_r\left(\frac{\eta^{\omega} t_r^{\omega-1}}{\Gamma(\omega)e^{\eta t_r}}\right)dt_r\right)\left\{\frac{ba^{b}}{c}\left[(a+\ln d-\ln x)^{-b}\right] + \sum_{i=2}^{N}\left\{\frac{ba^{b}}{c}\left[(a+\ln d-\ln([i-(i-1)\delta]x))^{-b}\right] - \frac{ba^{b}}{c}\left[(a+\ln d-\ln([i-1-(i-1)\delta]x))^{-b}\right]\right\}\right\}(Nx)^{-1}.$$

The posterior distribution for α and β, incorporating the likelihood of the observed breakdown times $t_1, t_2, \ldots, t_n$, is given by $f'(\alpha,\beta) \propto L(t_1,t_2,\ldots,t_n|\alpha,\beta)f(\alpha,\beta)$.

By applying Bayes' theorem, the posterior distribution becomes:

$$f'(\alpha,\beta) \propto K'\alpha^{b+n-1}\beta^{b+n-1}\left(e^{-a}d^{b}\prod_{i=1}^{n} t_i\right)^{\beta-1}e^{-\alpha\left(cd^{\beta}+t_n^{\beta}\right)}$$

This expression represents the updated belief about α and β after incorporating the observed data, with the prior knowledge being refined by the information gained from the sample. The term K' is a normalizing constant, and the exponential and product terms account for the relationship between the observed data and the parameters.

COST EVALUATION: In this study, the system's breakdown process is modeled as an NHPP with a power law intensity function. Assuming that expert opinions and historical data provide a suitable prior distribution for conducting Bayesian analysis, the expected total cost per unit time for the prior and posterior analyses can be reformulated from the following equations:

$$E[C(N)] = \frac{\sum_{i=1}^{N-1} C_{pm_i} + C_{rp} + \left(C_{mr} + C_{pl}\int_{\varphi}^{\infty} t_r\left(\frac{\eta^{\omega} t_r^{\omega-1}}{\Gamma(\omega)e^{\eta t_r}}\right)dt_r\right)\left\{E\left[\alpha(t_1^{-})^{\beta}\right] + \sum_{i=2}^{N}\left\{E\left[\alpha(t_i^{-})^{\beta}\right] - E\left[\alpha(t_{i-1}^{+})^{\beta}\right]\right\}\right\}}{Nx}$$

$$E'[C(N)] = \frac{\sum_{i=1}^{N-1} C_{pm_i} + C_{rp} + \left(C_{mr} + C_{pl}\int_{\varphi}^{\infty} t_r\left(\frac{\eta^{\omega} t_r^{\omega-1}}{\Gamma(\omega)e^{\eta t_r}}\right)dt_r\right)\left\{E'\left[\alpha(t_1^{-})^{\beta}\right] + \sum_{i=2}^{N}\left\{E'\left[\alpha(t_i^{-})^{\beta}\right] - E'\left[\alpha(t_{i-1}^{+})^{\beta}\right]\right\}\right\}}{Nx}$$

ANALYTICAL FRAMEWORK:

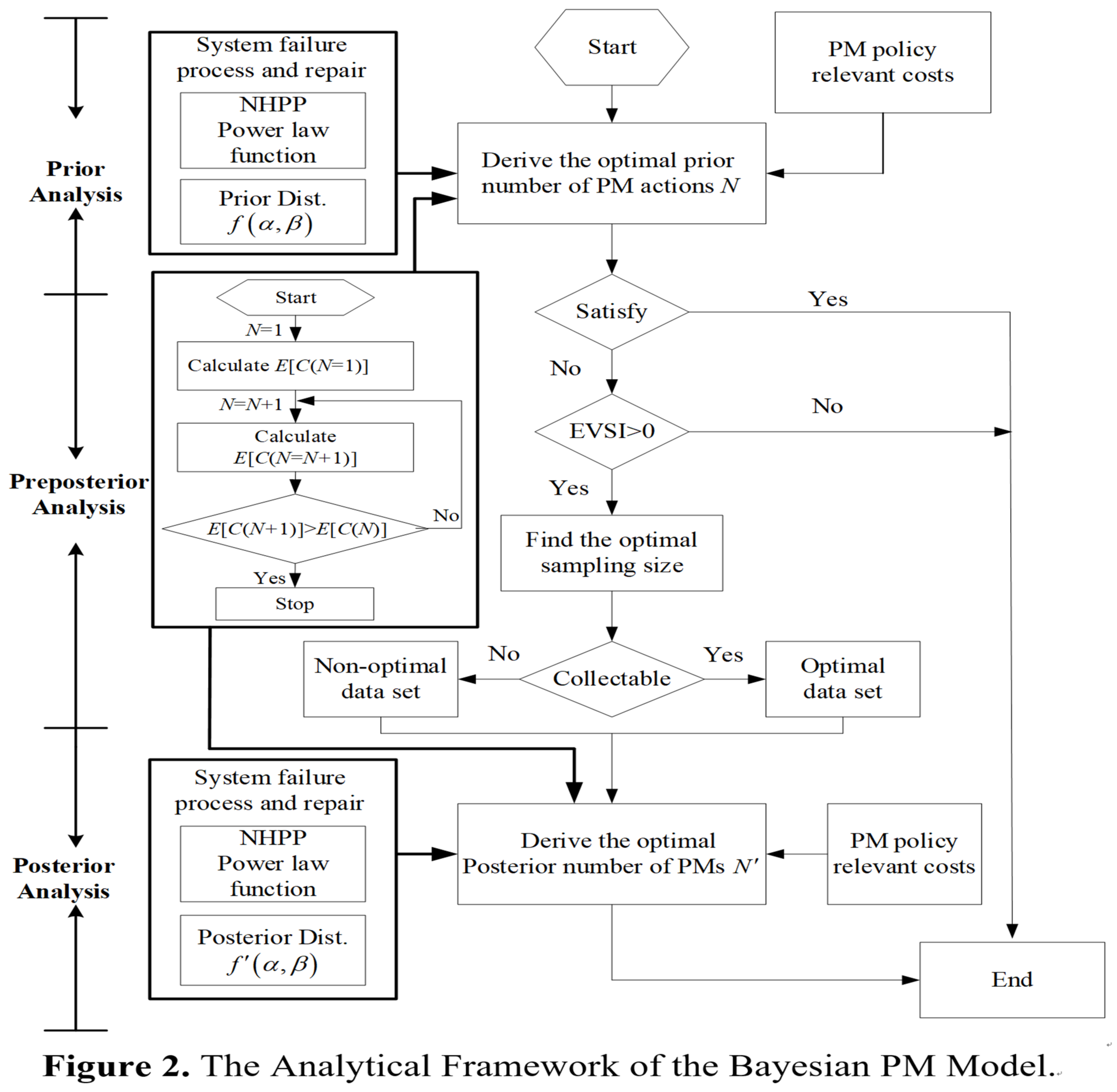

Figure 2. The Analytical Framework of the Bayesian PM Model.

REFERENCES

1. Huang Y S, Fang C C, Kuo C Y. A study on two-dimensional warranty with a preventive maintenance policy under burn-in or run-in[J]. Applied Stochastic Models in Business and Industry, 2024.
2. Fang C C, Hsu C C, Liu J H. The decision-making for the optimization of finance lease with facilities' two-dimensional deterioration[J]. Systems, 2022, 10(6): 210.